Technical Appendix

FORECASTING SOUTH AFRICAN MACROECONOMIC VARIABLES WITH A MARKOV-SWITCHING SMALL OPEN-ECONOMY DYNAMIC STOCHASTIC GENERAL EQUILIBRIUM MODEL

Mehmet Balcilar*†, Rangan Gupta† & Kevin Kotzé‡

April 2016

1 Model Solution

The method for deriving the model solution follows Maih (2015), which may be used to estimate endogenous transition probabilities for regime-switching parameters. It takes the form of a higher-order perturbation strategy that is able to approximate solutions for models that have both forward-looking and backward-looking variables.[1]

Alternative methods for solving these models are discussed in Davig and Leeper (2007), Davig *et al.* (2011), Farmer *et al.* (2008), Farmer *et al.* (2011), Foerster *et al.* (2014), and Svensson (2007). These methods include grid search techniques and perturbation methods that may be applied to linear and nonlinear models.

For the problem that we are looking to solve, we find that the method of Maih (2015) converges relatively rapidly on solutions that are both stable and unique (in the majority of the cases that we consider). As all the equations in the model have been linearised, we make use of first-order Markov chains that have constant transition probabilities for two regimes. This model includes both forward and backward looking variables and could be written as,

$$\mathbb{E}_t \left[A^{+}_{s_{t+1}} x_{t+1} \left(\bullet, s_t \right) \ ; \ A^{0}_{s_t} x_t \left(s_t, s_{t-1} \right) \ ; \ A^{-}_{s_t} x_{t-1} \left(s_{t-1}, s_{t-2} \right) \ ; \ B_{s_t} \varepsilon_t \right] = 0 \tag{1}$$

where x_t is a $n \times 1$ vector of endogenous variables and $\varepsilon_t \sim N\left(0, \varsigma_\vartheta\right)$ is the vector of exogenous shocks. The stochastic regime index s_t switches between two possibilities, such that $s_t = 1, 2$. These probabilities are assumed to be constant in this model, where s_t denotes the state of the system today and s_{t-1} denotes the state in the previous period. Therefore, the probability of moving from state s_t to state s_{t+1} in the next period is given by, $\mathcal{P}_{s_t, s_{t+1}} = prob\left(s_{t+1} | s_t\right)$. This allows for the expectation in the above expression to be described as,

$$\mathbb{E}_t \left[A^{+}_{s_{t+1}} x_{t+1} \left(\bullet, s_t \right) \right] \equiv \sum_{s_{t+1}=1}^{2} = \mathcal{P}_{s_t, s_{t+1}} \left[A_{s_{t+1}} \mathbb{E}_t x_{t+1} \left(s_{t+1}, s_t \right) \right] \tag{2}$$

The objective is then to derive a unique solution for this model, which will take the form

*Department of Economics, Eastern Mediterranean University, Famagusta, Turkish Republic of Northern Cyprus, via Mersin 10, Turkey; IPAG Business School, Paris, France.

†Department of Economics, University of Pretoria, Pretoria, 0002, South Africa.

‡School of Economics, Faculty of Commerce, University of Cape Town, Middle Campus, Private Bag, Rondebosch, 7701, South Africa.

[1] Maih (2015) provides an example of a model that is solved at the third-order, but notes that this technique could be applied to approximate a solution at even higher-orders.

$$x_t(s_t, s_{t-1}) = \mathcal{T}_{s_t} x_{t-1}(s_{t-1}, s_{t-2}) + R_{s_t}\varepsilon_t \quad (3)$$

As it would not be possible to derive an analytical solution for $\mathcal{T}_{s_t}$, we make use of a first-order perturbation technique that may be expressed as,

$$\mathcal{T}_{s_t}(\varepsilon) \simeq \mathcal{T}_{s_t}(\bar{\varepsilon}_{s_t}) + \mathcal{T}_{s_t}(\varepsilon - \bar{\varepsilon}_{s_t}) \quad (4)$$

Newton algorithms are then be employed to derive the solution for the above minimum state variable (MSV), where stable solutions for constant transition probability models employ the concept of mean square stability (MSS), which implies that all the eigenvalues are within the unit circle. A useful discussion regarding the use of MSV techniques and the application of the MSS criteria is contained in Farmer *et al.* (2009) and Farmer *et al.* (2011).[2]

2 Parameter Estimation

The likelihood function for this particular model would need to incorporate information that relates to the values of the observed variables, potential values of the unobserved variables, and the possible paths for each of the state variables (which are derived from Markov chains). To limit the amount of information that is carried forward at each iteration of the Kalman filter, Kim and Nelson (1999) take the average of all the possible state variables, when proceeding to the next iteration. In addition, Sims *et al.* (2008) suggest that one is able to derive computational savings by aggregating over all possible states after completing the prediction step, rather than after the updating step of the Kalman filter. We make use of this approach when deriving values for the unobserved processes.

The transition probabilities are then computed as per the method that is described in Hamilton (1989; 1994), while the smoother follows the Sims *et al.* (2008) procedure (which is based on the techniques of Kim and Nelson (1999) and Durbin and Koopman (2012)). Bayesian techniques are used to derive the final parameter estimates, so the likelihood function is combined with the prior distributions to derive the mode for the posterior estimates. To maximize the likelihood function, a grid-search technique is used to derive appropriate starting values, before Newton-based optimization techniques are employed to find the maxima. When reporting on the results of the estimation procedure, we provide details of the estimates for the mode and standard deviations. In addition, we also generate simulated posterior distributions with the aid of a Markov Chain Monte Carlo (MCMC) procedure, for selected models, where we make use of a single chain of 100,000 simulations. As this procedure is relatively time consuming, we only make use of the mode of the posterior estimates in the recursive forecasting exercise.

[2]See Maih (2015) for further details on the derivation of the iterative Newton algorithm that was used in this study.

3 Smoothed Transition Probabilities - Policy Parameters

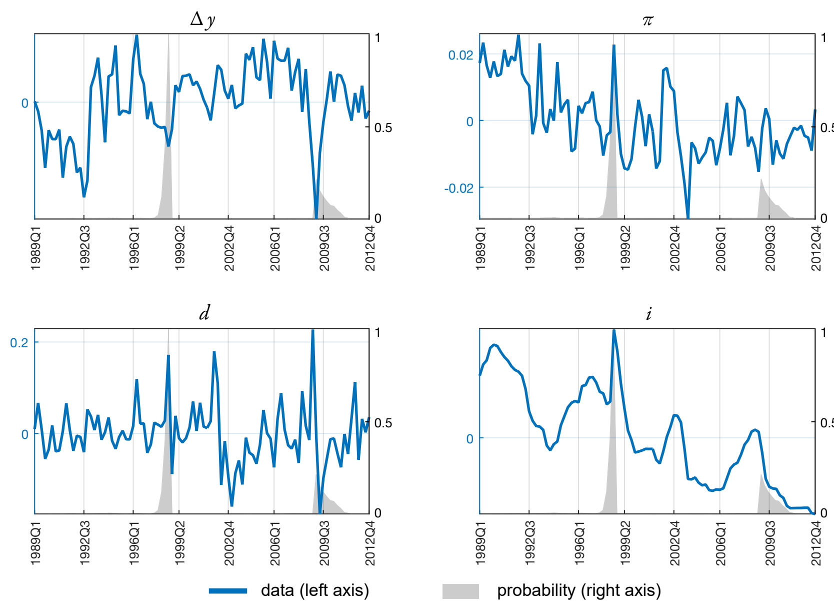

Figure 1: Regime-switching in monetary policy rule (only)

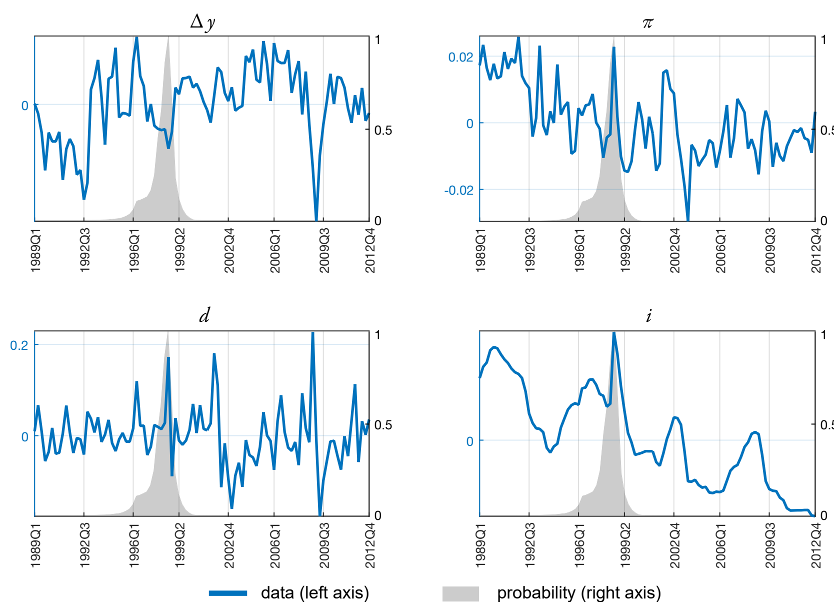

Figure 2: Regime-switching in monetary policy rule and variance of all shocks

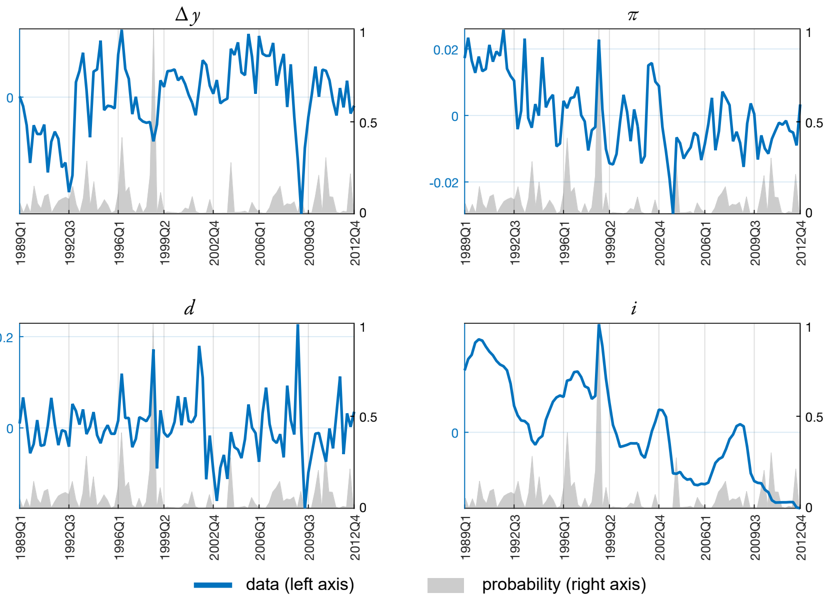

Figure 3: Regime-switching in monetary policy rule and variance of risk-premium shock

4 Smoothed Transition Probabilities - Variance of Shocks

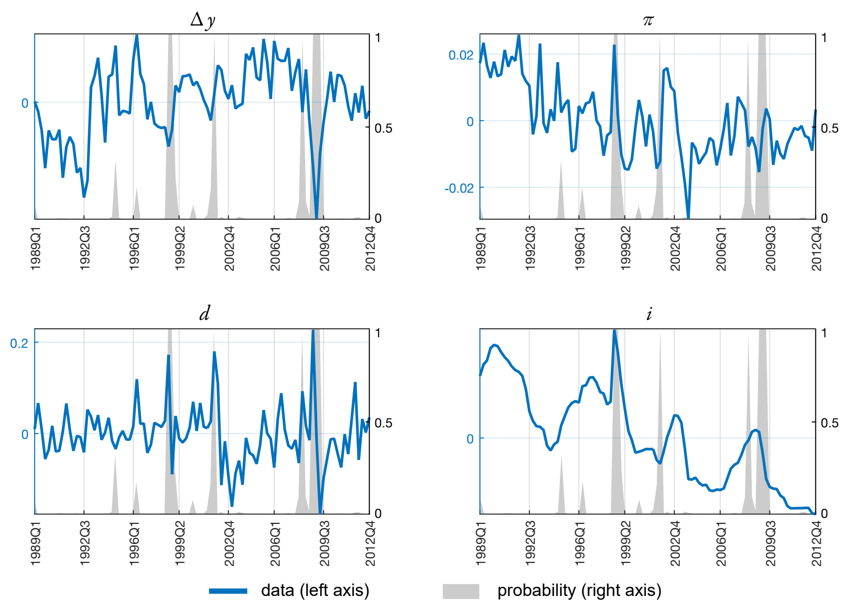

Figure 4: Regime-switching in variance of all shocks (only)

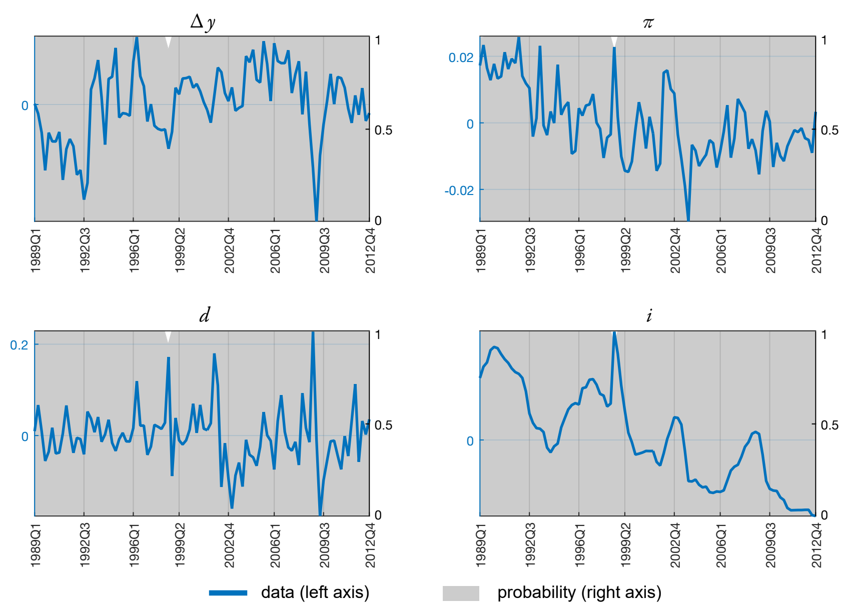

Figure 5: Regime-switching in monetary policy rule and variance of all shocks

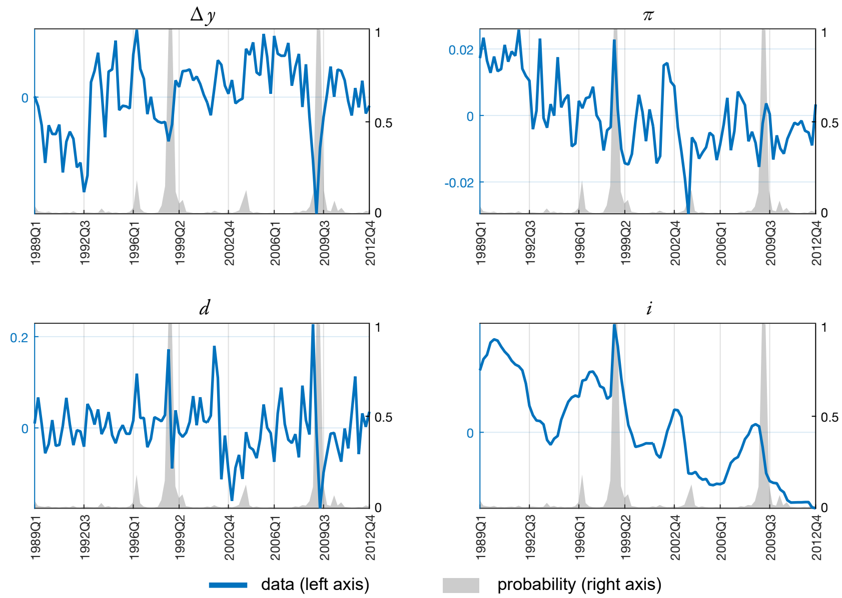

Figure 6: Regime-switching in variance of risk-premium shock (only)

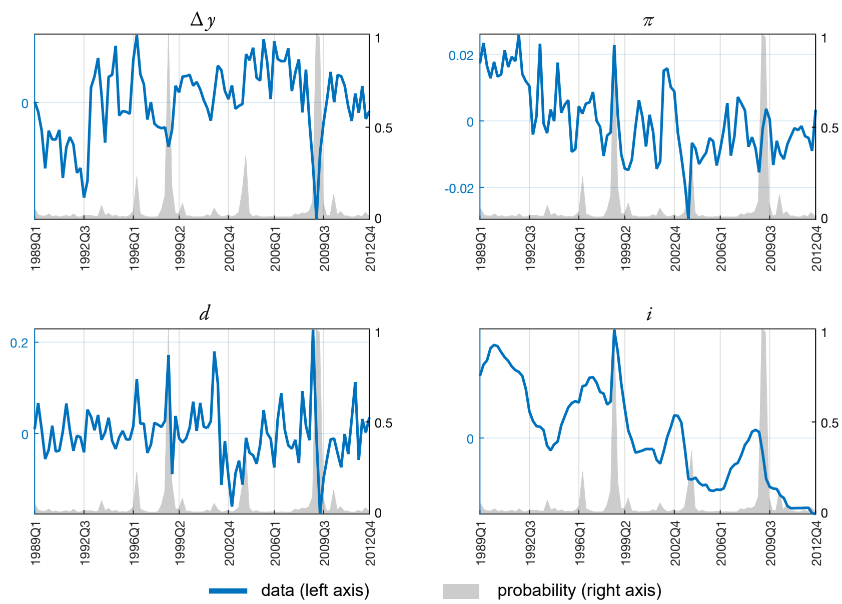

Figure 7: Regime-switching in monetary policy rule and variance of risk-premium shock

5 Posterior Parameter Estimates

Parameter	Distribution	Prior Mean	Prior Std.	Post Mode	Post Std.
ζ	beta	0.7	0.1	0.91	0.02
σ	gamma	1.5	0.37	1.14	0.27
ϑ	gamma	2	0.75	3.97	0.92
η	gamma	1.5	0.25	0.49	0.02
χ	normal	0.01	0	0.01	0.00
ϕ	beta	0.28	0.17	0.30	0.02
θ_h	beta	0.5	0.09	0.42	0.05
θ_f	beta	0.5	0.09	0.36	0.03
θ_w	beta	0.5	0.09	0.69	0.04
φ_h	beta	0.7	0.05	0.68	0.05
φ_f	beta	0.7	0.05	0.67	0.05
φ_w	beta	0.7	0.05	0.71	0.05
ρ_z	beta	0.6	0.1	0.97	0.01
ρ_c	beta	0.6	0.1	0.88	0.00
ρ_h	beta	0.6	0.1	0.91	0.02
ρ_f	beta	0.6	0.1	0.95	0.01
ρ_w	beta	0.6	0.1	0.27	0.06
ρ_d	beta	0.6	0.1	0.88	0.05
ρ_i	beta	0.6	0.1	0.41	0.06
ρ_{y*}	beta	0.6	0.1	0.95	0.01
$\rho_{\pi*}$	beta	0.6	0.1	0.60	0.06
ρ_{i*}	beta	0.6	0.1	0.94	0.01
ρ	beta	0.75	0.1	0.82	0.02
ϱ_π	gamma	1.5	0.25	1.62	0.21
ϱ_y	gamma	0.25	0.12	0.57	0.20
ϱ_d	gamma	0.12	0.05	0.06	0.02
ς^z	weibull	0.23	0.29	0.01	0.00
ς^c	weibull	0.23	0.29	0.00	0.00
ς^h	weibull	0.23	0.29	0.02	0.00
ς^f	weibull	0.23	0.29	0.07	0.01
ς^w	weibull	0.23	0.29	0.02	0.00
ς^d	weibull	0.23	0.29	0.00	0.00
ς^i	weibull	0.23	0.29	0.00	0.00
ς^{y*}	weibull	0.23	0.29	0.01	0.00
$\varsigma^{\pi*}$	weibull	0.23	0.29	0.00	0.00
ς^{i*}	weibull	0.23	0.29	0.00	0.00

Table 1: No-switching − parameter estimates

Parameter	Distribution	Prior Mean	Prior Std.	Post Mode	Post Std.
ζ	beta	0.7	0.1	0.91	0.02
σ	gamma	1.5	0.37	1.2	0.27
ϑ	gamma	2	0.75	4.36	0.94
η	gamma	1.5	0.25	0.49	0.02
χ	normal	0.01	0	0.01	0.00
ϕ	beta	0.28	0.17	0.32	0.01
θ_h	beta	0.5	0.09	0.42	0.05
θ_f	beta	0.5	0.09	0.35	0.03
θ_w	beta	0.5	0.09	0.66	0.03
φ_h	beta	0.7	0.05	0.68	0.05
φ_f	beta	0.7	0.05	0.68	0.05
φ_w	beta	0.7	0.05	0.71	0.05
ρ_z	beta	0.6	0.1	0.97	0.01
ρ_c	beta	0.6	0.1	0.88	0.00
ρ_h	beta	0.6	0.1	0.92	0.02
ρ_f	beta	0.6	0.1	0.95	0.01
ρ_w	beta	0.6	0.1	0.27	0.06
ρ_d	beta	0.6	0.1	0.88	0.05
ρ_i	beta	0.6	0.1	0.44	0.06
ρ_{y*}	beta	0.6	0.1	0.95	0.01
$\rho_{\pi*}$	beta	0.6	0.1	0.6	0.06
ρ_{i*}	beta	0.6	0.1	0.94	0.01
ρ $(\kappa = 1)$	beta	0.75	0.1	0.7	0.07
ρ $(\kappa = 2)$	beta	0.75	0.1	0.9	0.02
ϱ_π $(\kappa = 1)$	gamma	1.52	0.51	2.2	0.79
ϱ_π $(\kappa = 2)$	gamma	1.52	0.51	2.16	0.41
ϱ_y $(\kappa = 1)$	gamma	0.28	0.47	0.00	0.00
ϱ_y $(\kappa = 2)$	gamma	0.28	0.47	0.00	0.00
ϱ_d $(\kappa = 1)$	gamma	0.14	0.23	0.00	0.00
ϱ_d $(\kappa = 2)$	gamma	0.14	0.23	0.00	0.00
ν^{1-2}	uniform	0	1	0.25	0.34
ν^{2-1}	uniform	0	1	0.02	0.03
ς^z	weibull	0.23	0.29	0.01	0.00
ς^c	weibull	0.23	0.29	0.00	0.00
ς^h	weibull	0.23	0.29	0.02	0.00
ς^f	weibull	0.23	0.29	0.08	0.01
ς^w	weibull	0.23	0.29	0.02	0.00
ς^d	weibull	0.23	0.29	0.00	0.00
ς^i	weibull	0.23	0.29	0.00	0.00
ς^{y*}	weibull	0.23	0.29	0.01	0.00
$\varsigma^{\pi*}$	weibull	0.23	0.29	0.00	0.00
ς^{i*}	weibull	0.23	0.29	0.00	0.00

Table 2: Switching in policy − parameter estimates

Parameter	Distribution	Prior Mean	Prior Std.	Post Mode	Post Std.
ζ	beta	0.7	0.1	0.93	0.02
σ	gamma	1.5	0.37	0.92	0.23
ϑ	gamma	2	0.75	3.97	0.91
η	gamma	1.5	0.25	0.48	0.02
χ	normal	0.01	0	0.01	0.00
ϕ	beta	0.28	0.17	0.51	0.03
θ_h	beta	0.5	0.09	0.44	0.06
θ_f	beta	0.5	0.09	0.69	0.06
θ_w	beta	0.5	0.09	0.65	0.04
φ_h	beta	0.7	0.05	0.69	0.05
φ_f	beta	0.7	0.05	0.64	0.06
φ_w	beta	0.7	0.05	0.68	0.05
ρ_z	beta	0.6	0.1	0.97	0.01
ρ_c	beta	0.6	0.1	0.58	0.15
ρ_h	beta	0.6	0.1	0.91	0.03
ρ_f	beta	0.6	0.1	0.37	0.12
ρ_w	beta	0.6	0.1	0.27	0.06
ρ_d	beta	0.6	0.1	0.44	0.08
ρ_i	beta	0.6	0.1	0.49	0.06
ρ_{y*}	beta	0.6	0.1	0.96	0.01
$\rho_{\pi*}$	beta	0.6	0.1	0.6	0.07
ρ_{i*}	beta	0.6	0.1	0.95	0.01
ρ	beta	0.75	0.1	0.91	0.02
ϱ_π	gamma	1.5	0.25	1.21	0.25
ϱ_y	gamma	0.25	0.12	0.33	0.17
ϱ_d	gamma	0.12	0.05	0.06	0.02
$\varsigma^z(\vartheta = 1)$	weibull	0.23	0.29	0.01	0.00
$\varsigma^z(\vartheta = 2)$	weibull	0.23	0.29	0.01	0.00
$\varsigma^c(\vartheta = 1)$	weibull	0.23	0.29	0.00	0.00
$\varsigma^c(\vartheta = 2)$	weibull	0.23	0.29	0.01	0.00
$\varsigma^h(\vartheta = 1)$	weibull	0.23	0.29	0.02	0.00
$\varsigma^h(\vartheta = 2)$	weibull	0.23	0.29	0.01	0.00
$\varsigma^f(\vartheta = 1)$	weibull	0.23	0.29	0.03	0.00
$\varsigma^f(\vartheta = 2)$	weibull	0.23	0.29	0.05	0.01
$\varsigma^w(\vartheta = 1)$	weibull	0.23	0.29	0.02	0.00
$\varsigma^w(\vartheta = 2)$	weibull	0.23	0.29	0.02	0.01
$\varsigma^d(\vartheta = 1)$	weibull	0.23	0.29	0.02	0.00
$\varsigma^d(\vartheta = 2)$	weibull	0.23	0.29	0.06	0.02
$\varsigma^i(\vartheta = 1)$	weibull	0.23	0.29	0.00	0.00
$\varsigma^i(\vartheta = 2)$	weibull	0.23	0.29	0.01	0.00
$\varsigma^{y*}(\vartheta = 1)$	weibull	0.23	0.29	0.01	0.00
$\varsigma^{y*}(\vartheta = 2)$	weibull	0.23	0.29	0.01	0.00
$\varsigma^{\pi*}(\vartheta = 1)$	weibull	0.23	0.29	0.00	0.00
$\varsigma^{\pi*}(\vartheta = 2)$	weibull	0.23	0.29	0.00	0.00
$\varsigma^{i*}(\vartheta = 1)$	weibull	0.23	0.29	0.00	0.00
$\varsigma^{i*}(\vartheta = 2)$	weibull	0.23	0.29	0.00	0.00
ω^{1-2}	uniform	0	1	0.08	0.03
ω^{2-1}	uniform	0	1	0.43	0.23

Table 3: Switching in volatility − parameter estimates

Parameter	Distribution	Prior Mean	Prior Std.	Post Mode	Post Std.
ζ	beta	0.7	0.1	0.67	0.03
σ	gamma	1.5	0.37	1.06	0.03
ϑ	gamma	2	0.75	2.22	0.11
η	gamma	1.5	0.25	0.47	0.01
χ	normal	0.01	0	0.01	0.00
ϕ	beta	0.28	0.17	0.09	0.00
θ_h	beta	0.5	0.09	0.49	0.02
θ_f	beta	0.5	0.09	0.27	0.01
θ_w	beta	0.5	0.09	0.75	0.03
φ_h	beta	0.7	0.05	0.65	0.04
φ_f	beta	0.7	0.05	0.68	0.04
φ_w	beta	0.7	0.05	0.75	0.03
ρ_z	beta	0.6	0.1	0.97	0.01
ρ_c	beta	0.6	0.1	0.90	0.00
ρ_h	beta	0.6	0.1	0.96	0.01
ρ_f	beta	0.6	0.1	0.92	0.01
ρ_w	beta	0.6	0.1	0.33	0.01
ρ_d	beta	0.6	0.1	0.97	0.01
ρ_i	beta	0.6	0.1	0.28	0.03
ρ_{y*}	beta	0.6	0.1	0.94	0.01
$\rho_{\pi*}$	beta	0.6	0.1	0.48	0.00
ρ_{i*}	beta	0.6	0.1	0.91	0.02
$\rho(\kappa=1)$	beta	0.75	0.1	0.7	0.04
$\rho(\kappa=2)$	beta	0.75	0.1	0.89	0.00
$\varrho_\pi(\kappa=1)$	gamma	1.52	0.51	2.5	0.15
$\varrho_\pi(\kappa=2)$	gamma	1.52	0.51	2.5	0.07
$\varrho_y(\kappa=1)$	gamma	0.28	0.47	0.00	0.00
$\varrho_y(\kappa=2)$	gamma	0.28	0.47	2.16	0.00
$\varrho_d(\kappa=1)$	gamma	0.14	0.23	0.00	0.00
$\varrho_d(\kappa=2)$	gamma	0.14	0.23	0.00	0.00
ν^{1-2}	uniform	0	1	0.00	0.00
ν^{2-1}	uniform	0	1	0.00	0.00
$\varsigma^z(\vartheta=1)$	weibull	0.23	0.29	0.01	0.00
$\varsigma^z(\vartheta=2)$	weibull	0.23	0.29	0.01	0.01
$\varsigma^c(\vartheta=1)$	weibull	0.23	0.29	0.00	0.00
$\varsigma^c(\vartheta=2)$	weibull	0.23	0.29	0.01	0.01
$\varsigma^h(\vartheta=1)$	weibull	0.23	0.29	0.01	0.00
$\varsigma^h(\vartheta=2)$	weibull	0.23	0.29	0.04	0.03
$\varsigma^f(\vartheta=1)$	weibull	0.23	0.29	0.10	0.01
$\varsigma^f(\vartheta=2)$	weibull	0.23	0.29	0.04	0.02
$\varsigma^w(\vartheta=1)$	weibull	0.23	0.29	0.01	0.00
$\varsigma^w(\vartheta=2)$	weibull	0.23	0.29	0.00	0.01
$\varsigma^d(\vartheta=1)$	weibull	0.23	0.29	0.00	0.00
$\varsigma^d(\vartheta=2)$	weibull	0.23	0.29	0.07	0.04
$\varsigma^i(\vartheta=1)$	weibull	0.23	0.29	0.00	0.00
$\varsigma^i(\vartheta=2)$	weibull	0.23	0.29	0.02	0.06
$\varsigma^{y*}(\vartheta=1)$	weibull	0.23	0.29	0.01	0.00
$\varsigma^{y*}(\vartheta=2)$	weibull	0.23	0.29	0.01	0.00
$\varsigma^{\pi*}(\vartheta=1)$	weibull	0.23	0.29	0.00	0.00
$\varsigma^{\pi*}(\vartheta=2)$	weibull	0.23	0.29	0.00	0.00
$\varsigma^{i*}(\vartheta=1)$	weibull	0.23	0.29	0.00	0.00
$\varsigma^{i*}(\vartheta=2)$	weibull	0.23	0.29	0.20	0.05
ω^{1-2}	uniform	0	1	0.02	0.02
ω^{2-1}	uniform	0	1	1.00	0.02

Table 4: Switching in policy and volatility − parameter estimates

Parameter	Distribution	Prior Mean	Prior Std.	Post Mode	Post Std.
ζ	beta	0.7	0.1	0.86	0.00
σ	gamma	1.5	0.37	1.01	0.23
ϑ	gamma	2	0.75	3.00	0.59
η	gamma	1.5	0.25	0.46	0.01
χ	normal	0.01	0	0.01	0.00
ϕ	beta	0.28	0.17	0.30	0.02
θ_h	beta	0.5	0.09	0.42	0.05
θ_f	beta	0.5	0.09	0.34	0.03
θ_w	beta	0.5	0.09	0.68	0.02
φ_h	beta	0.7	0.05	0.68	0.05
φ_f	beta	0.7	0.05	0.68	0.05
φ_w	beta	0.7	0.05	0.71	0.05
ρ_z	beta	0.6	0.1	0.96	0.01
ρ_c	beta	0.6	0.1	0.86	0.00
ρ_h	beta	0.6	0.1	0.94	0.02
ρ_f	beta	0.6	0.1	0.93	0.01
ρ_w	beta	0.6	0.1	0.26	0.06
ρ_d	beta	0.6	0.1	0.95	0.02
ρ_i	beta	0.6	0.1	0.49	0.00
ρ_{y*}	beta	0.6	0.1	0.95	0.01
$\rho_{\pi*}$	beta	0.6	0.1	0.61	0.06
ρ_{i*}	beta	0.6	0.1	0.93	0.02
ρ	beta	0.75	0.1	0.85	0.02
ϱ_π	gamma	1.5	0.25	1.80	0.20
ϱ_y	gamma	0.25	0.12	0.64	0.22
ϱ_d	gamma	0.12	0.05	0.06	0.02
ς^z	weibull	0.23	0.29	0.01	0.00
ς^c	weibull	0.23	0.29	0.00	0.00
ς^h	weibull	0.23	0.29	0.02	0.00
ς^f	weibull	0.23	0.29	0.08	0.01
ς^w	weibull	0.23	0.29	0.02	0.00
ς^i	weibull	0.23	0.29	0.00	0.00
ς^{y*}	weibull	0.23	0.29	0.01	0.00
$\varsigma^{\pi*}$	weibull	0.23	0.29	0.00	0.00
ς^{i*}	weibull	0.23	0.29	0.00	0.00
$\varsigma^d(\vartheta = 1)$	weibull	0.23	0.29	0.00	0.00
$\varsigma^d(\vartheta = 2)$	weibull	0.23	0.29	0.01	0.00
ω^{1-2}	uniform	0	1	0.04	0.03
ω^{2-1}	uniform	0	1	0.50	0.25

Table 5: Switching in risk-premium volatility − parameter estimates

Parameter	Distribution	Prior Mean	Prior Std.	Post Mode	Post Std.
ζ	beta	0.7	0.1	0.83	0.00
σ	gamma	1.5	0.37	0.65	0.00
ϑ	gamma	2	0.75	1.24	0.29
η	gamma	1.5	0.25	0.44	0.00
χ	normal	0.01	0	0.01	0.00
ϕ	beta	0.28	0.17	0.25	0.02
θ_h	beta	0.5	0.09	0.45	0.04
θ_f	beta	0.5	0.09	0.3	0.03
θ_w	beta	0.5	0.09	0.79	0.04
φ_h	beta	0.7	0.05	0.68	0.05
φ_f	beta	0.7	0.05	0.68	0.02
φ_w	beta	0.7	0.05	0.73	0.02
ρ_z	beta	0.6	0.1	0.97	0.01
ρ_c	beta	0.6	0.1	0.86	0.00
ρ_h	beta	0.6	0.1	0.93	0.02
ρ_f	beta	0.6	0.1	0.94	0.01
ρ_w	beta	0.6	0.1	0.26	0.06
ρ_d	beta	0.6	0.1	0.95	0.02
ρ_i	beta	0.6	0.1	0.40	0.00
ρ_{y*}	beta	0.6	0.1	0.94	0.01
$\rho_{\pi*}$	beta	0.6	0.1	0.55	0.06
ρ_{i*}	beta	0.6	0.1	0.93	0.01
$\rho(\kappa = 1)$	beta	0.75	0.1	0.81	0.11
$\rho(\kappa = 2)$	beta	0.75	0.1	0.85	0.01
$\varrho_\pi(\kappa = 1)$	gamma	1.52	0.51	1.04	0.35
$\varrho_\pi(\kappa = 2)$	gamma	1.52	0.51	2.47	0.00
$\varrho_y(\kappa = 1)$	gamma	0.28	0.47	0.00	0.00
$\varrho_y(\kappa = 2)$	gamma	0.28	0.47	1.97	0.06
$\varrho_d(\kappa = 1)$	gamma	0.14	0.23	0.00	0.00
$\varrho_d(\kappa = 2)$	gamma	0.14	0.23	0.00	0.00
ν^{1-2}	uniform	0	1	0.00	0.00
ν^{2-1}	uniform	0	1	0.00	0.00
ς^z	weibull	0.23	0.29	0.01	0.00
ς^c	weibull	0.23	0.29	0.00	0.00
ς^h	weibull	0.23	0.29	0.01	0.00
ς^f	weibull	0.23	0.29	0.09	0.02
ς^w	weibull	0.23	0.29	0.02	0.00
ς^i	weibull	0.23	0.29	0.00	0.00
ς^{y*}	weibull	0.23	0.29	0.01	0.00
$\varsigma^{\pi*}$	weibull	0.23	0.29	0.00	0.00
ς^{i*}	weibull	0.23	0.29	0.00	0.00
$\varsigma^d(\vartheta = 1)$	weibull	0.23	0.29	0.00	0.00
$\varsigma^d(\vartheta = 2)$	weibull	0.23	0.29	0.34	0.00
ω^{1-2}	uniform	0	1	0.01	0.01
ω^{2-1}	uniform	0	1	1.00	0.14

Table 6: Switching in policy and risk-premium volatility − parameter estimates

6 Posterior Parameter Distributions

The following posterior distributions were simulated with the aid of a Markov Chain Monte Carlo (MCMC) procedure. We include the results for the three models that provided the superior in-sample statistics. These include the following variants of the model:

- No regime-switching
- Regime-switching in the variance of the shocks (only)
- Regime-switching in the monetary policy rule and the variance of the shocks

To ease the comparison, the parameter estimates for each of these models are displayed in the separate column.

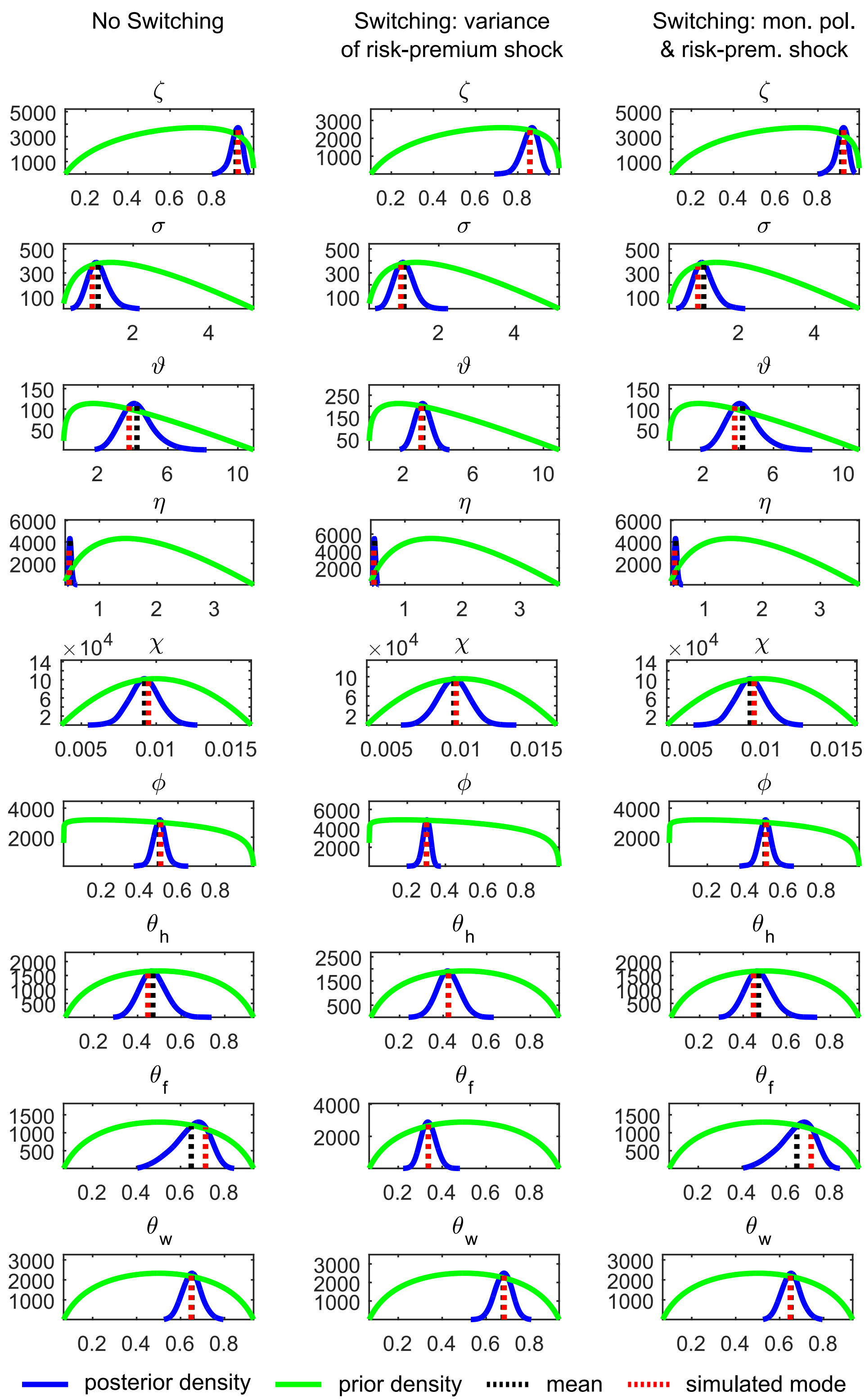

Figure 8: Posterior parameter estimates for selected models

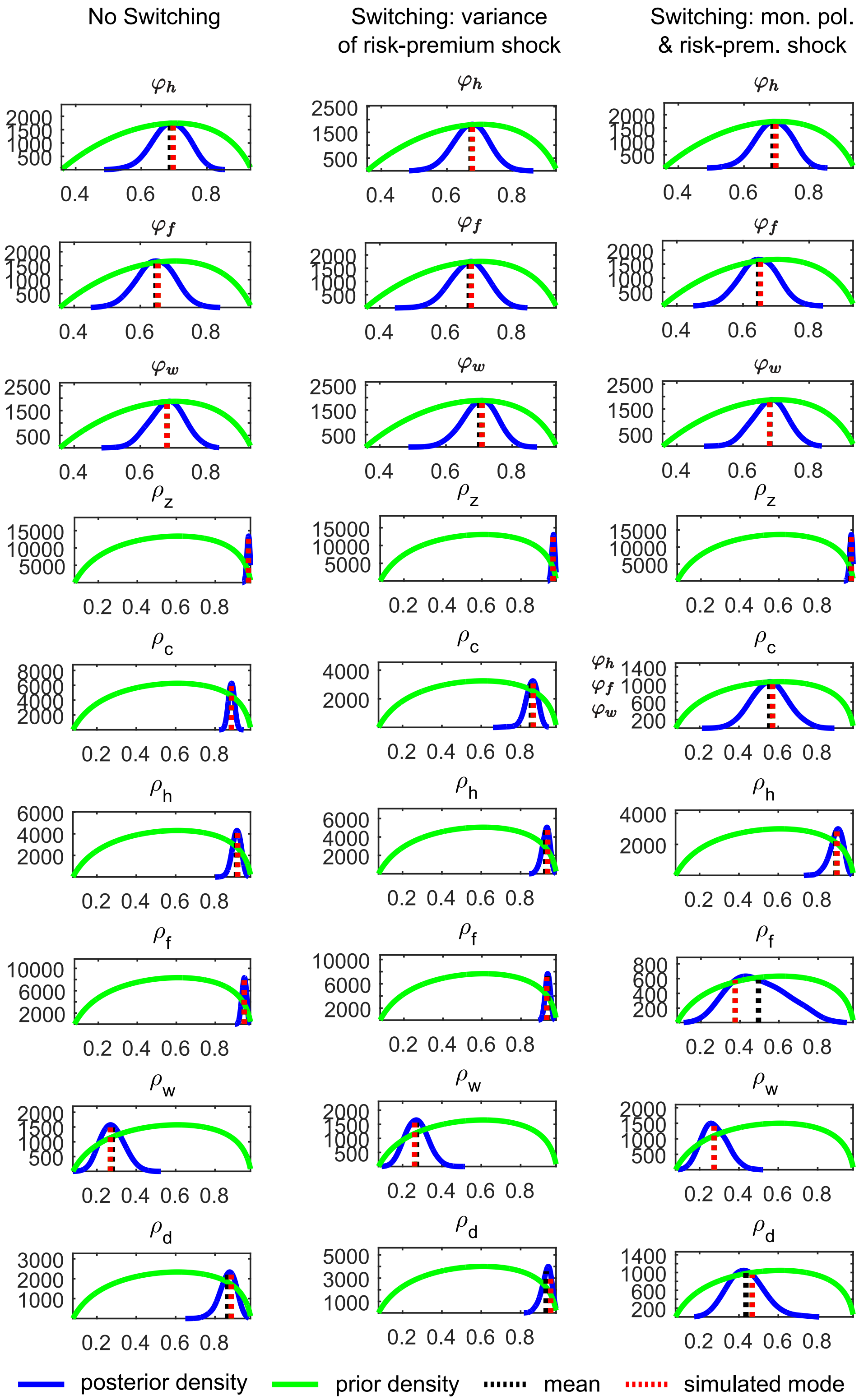

Figure 9: Posterior parameter estimates for selected models

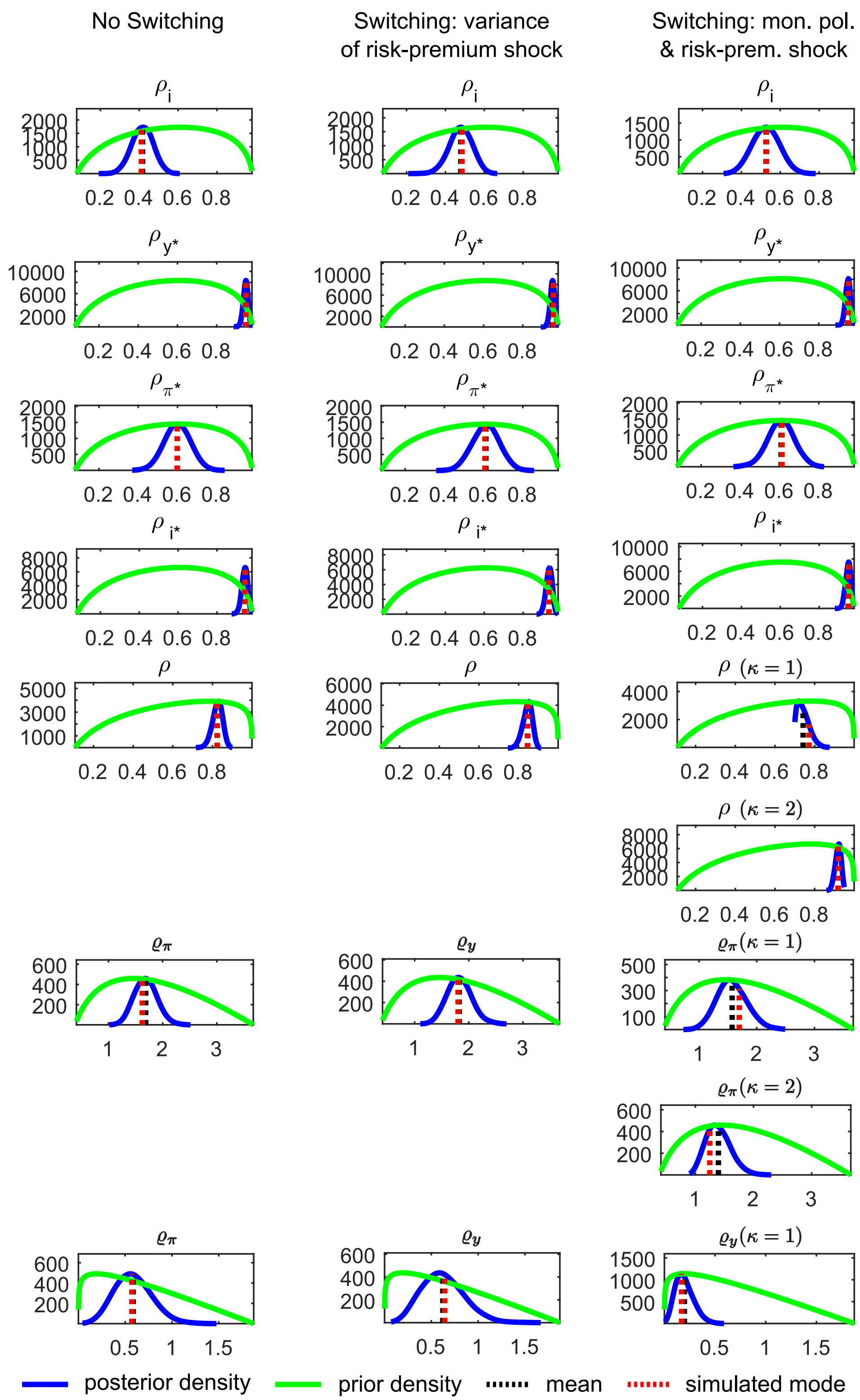

Figure 10: Posterior parameter estimates for selected models

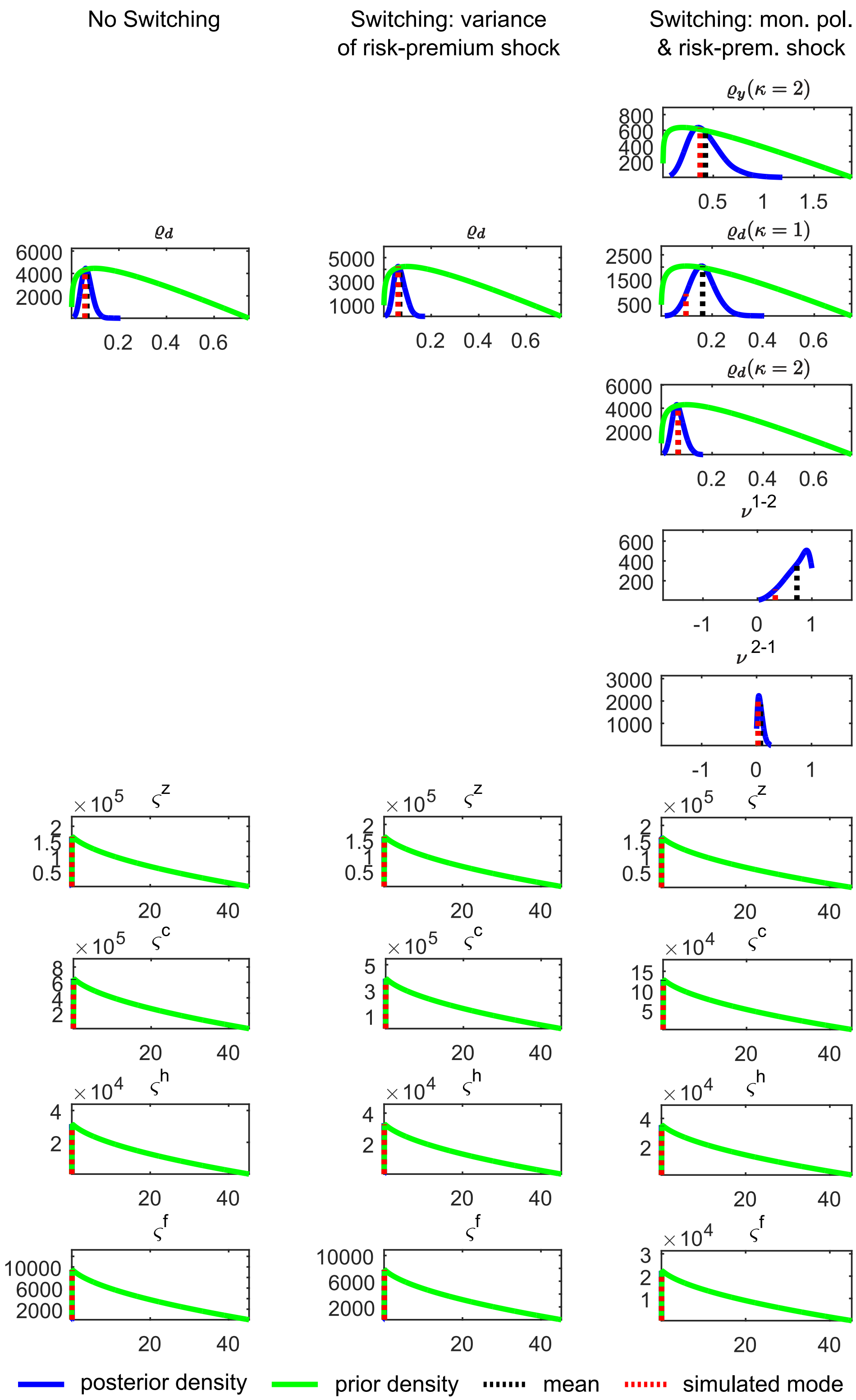

Figure 11: Posterior parameter estimates for selected models

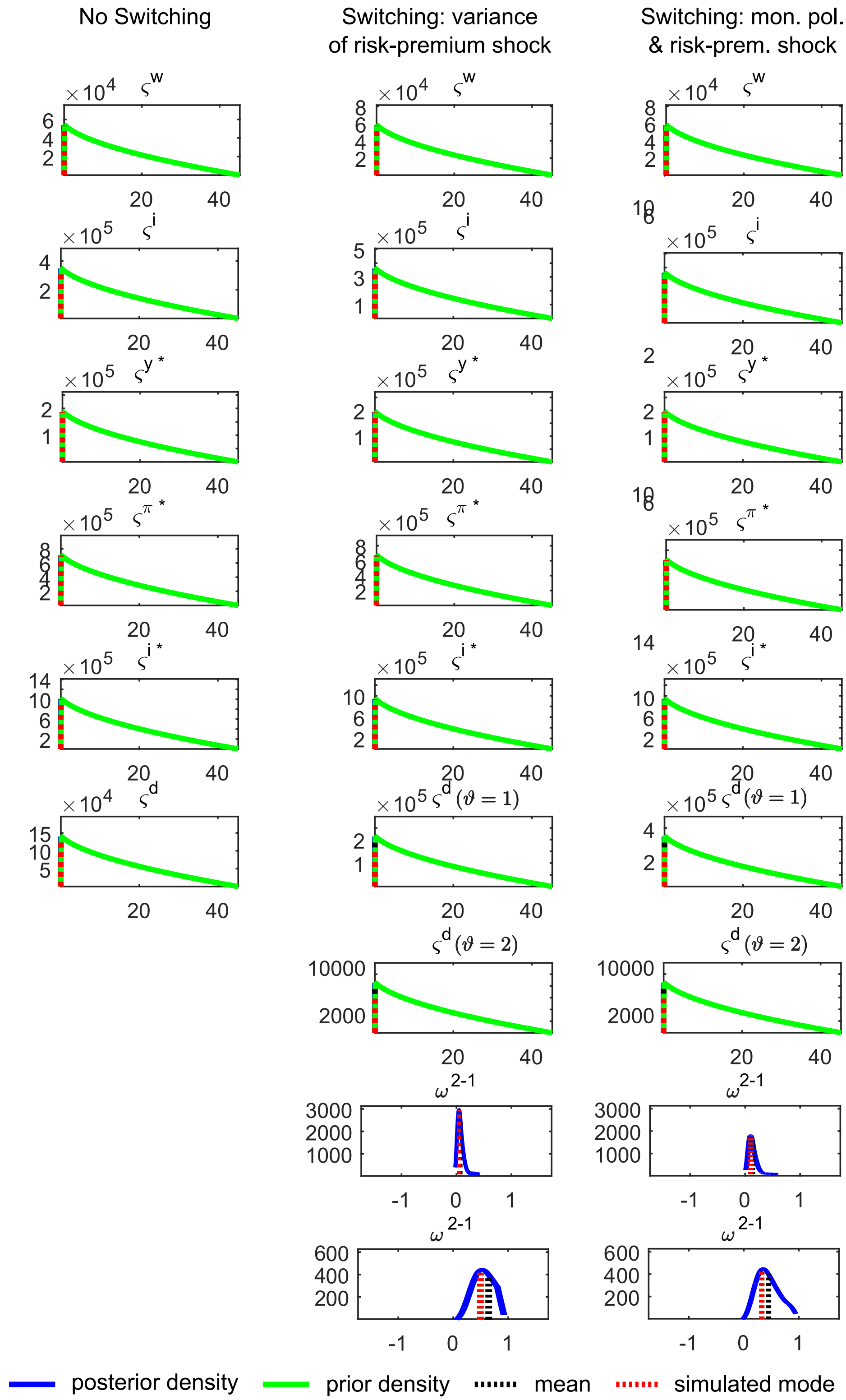

Figure 12: Posterior parameter estimates for selected models

7 Impulse Response Functions

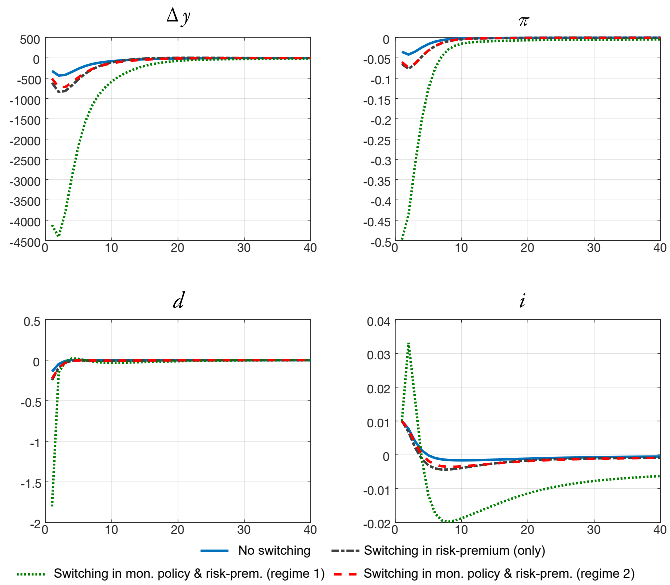

Figure 13: Impulse response function - monetary policy shock

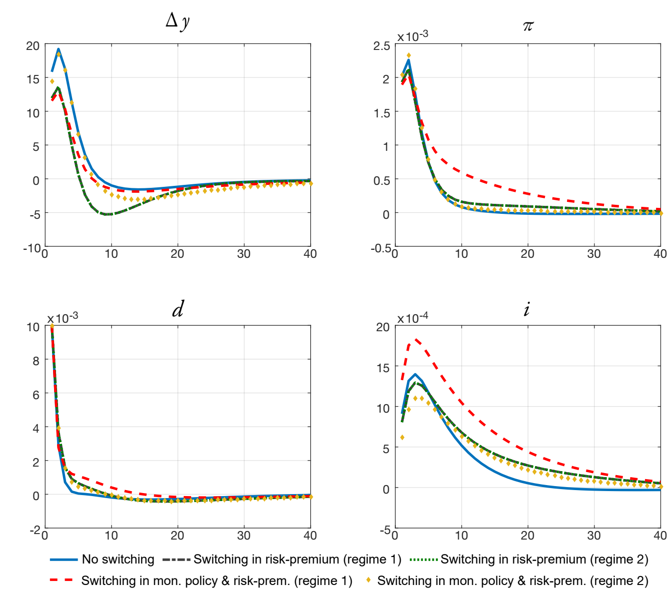

Figure 14: Impulse response function - risk-premium shock

References

Davig, T. and Leeper, E.M. 2007. Fluctuating Macro Policies and the Fiscal Theory. In D. Acemoglu, K. Rogoff and M. Woodford (eds.), *NBER Macroeconomics Annual 2006*, volume 21, pages 247–316. Cambridge, Ma.: National Bureau of Economic Research.

Davig, T., Leeper, E.M. and Walker, T.B. 2011. Inflation and the Fiscal Limit. *European Economic Review*, 55(1):31–47.

Durbin, J. and Koopman, S.J. 2012. *Time Series Analysis by State Space Methods*. 2nd edition. Oxford: Oxford University Press.

Farmer, R.E., Waggoner, D.F. and Zha, T. 2008. Minimal state variable solutions to Markov-switching rational expectations models. Technical Report Working Paper 2008-23, Federal Reserve Bank of Atlanta.

Farmer, R.E., Waggoner, D.F. and Zha, T. 2009. Generalizing the Taylor Principle: A Comment. Technical Report, UCLA Department of Economics.

Farmer, R.E., Waggoner, D.F. and Zha, T. 2011. Minimal state variable solutions to Markov-switching rational expectations models. *Journal of Economic Dynamics and Control*, 35(12):2150–2166.

Foerster, A., Rubio-Ramrez, J., Waggoner, D.F. and Zha, T. 2014. Perturbation Methods for Markov-Switching DSGE Models. NBER Working Papers 20390, National Bureau of Economic Research, Inc.

Hamilton, J.D. 1989. A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle. *Econometrica*, 57:357–384.

Hamilton, J.D. 1994. *Time Series Analysis*. Princeton: Princeton University Press.

Kim, C.-J. and Nelson, C.R. 1999. Has the US Economy Become More Stable? *The Review of Economics and Statistics*, 81:608–616.

Maih, J. 2015. Efficient Perturbation Methods for Solving Regime-Switching DSGE Models. Technical Report 01/2015, Norges Bank Research.

Sims, C.A., Waggoner, D.F. and Zha, T. 2008. Methods for inference in large multiple-equation Markov-switching models. *Journal of Econometrics*, 146(2):255–274.

Svensson, L.E.O. 2007. Optimal Inflation Targeting: Further Developments of Inflation Targeting.